Probabilistic seismic hazard analysis using an advanced intensity measure accounting for structural degradation

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ABSTRACT

Based on a conventional seismic hazard analysis, it is possible to estimate the annual probability of exceedance of a given ground motion parameter. Commonly it is used the spectral acceleration corresponding to the fundamental structural period $Sa(T_1)$, which is the ground motion intensity measure (*IM*) most used for probabilistic seismic hazard analysis (*PSHA*). However, $Sa(T_1)$ has some limitations because it does not consider the effect of the elongation of the vibration period of a structure due to non-linear structural behavior or to mechanical properties degradation. Consequently, advanced seismic *IMs* have been proposed with the aim to correct the inconveniences of traditional *IMs*. The primary objective of the present study is to perform a *PSHA* with a new ground motion *IM* called I_{Np}, which is based on $Sa(T_1)$ and a parameter that characterizes the spectral shape. For this aim, it is required to have correlation coefficients between spectral acceleration values at multiple periods. Seismic records from interplate events, registered in the firm ground of Mexico City are employed to compute the correlation coefficients. Using attenuation models, correlation coefficients and the methodology introduced in the present paper, it is possible to describe the complete distribution of the logarithm of I_{Np}; with this, *PSHA* is carried out. The results are presented by means uniform hazard spectra (*UHS*) of I_{Np}, and are compared with their corresponding $Sa(T_1)$ *UHS*. For firm ground there is not significantly different between both spectra; however, for the case of soft soil, there is an amplification of the order up to 25% in the spectral I_{NP} *UHS* ordinates with respect to those corresponding to $Sa(T_1)$.

Keywords: probabilistic seismic hazard analysis; ground-motion intensity measure; spectral correlation coefficients; degradation of structures.

INTRODUCTION

One of the primary objectives in earthquake engineering is to define the intensity of an expected seismic excitation; however, due to the uncertainty associated with the number, location and magnitude of future ground motions; the problem has been addressed through a probabilistic seismic hazard analysis (*PSHA*). A conventional *PSHA* estimates the mean annual rate of exceedance of a given seismic parameter; commonly it is used the spectral acceleration measured at the fundamental period of a structure $Sa(T_1)$. However, this ground motion intensity measure (*IM*) has some limitations, because it does not consider the period shift effect of a structure resulting from its non-linear behavior. Some researchers suggest using vector-valued ground motion *IMs*, which more accurate evaluations of seismic performance are achieved by including two or more parameters representative of the seismic event. For example, the ground motion *IM* $<Sa(T_1), R_{T1,T2}>$, derived from the scalar *IM* proposed by Cordova *et al*. [1], where $R_{T1,T2}$, is the ratio between the spectral acceleration at period T_1 and a longer period T_2. Baker and Cornell [2] developed the *IM* $<Sa(T_1), \varepsilon>$, where the parameter ε, is the number of standard deviations between the actual spectral acceleration and that calculated with an attenuation function. Similarly, Tothong and Luco [3] presented two intensity measurements based on the inelastic spectral displacement for structures dominated by their first mode of vibration and for structures sensitive to their higher modes. Bojórquez and Iervolino [4] developed the intensity measure $<Sa(T_1), Np>$, where Np is a parameter proxy for the spectral shape, showing that this measure exhibits an improvement in predicting the seismic response in comparison with other *IMs*. However, a *PSHA* with vector-valued *IMs* is complicated and impractical. Consequently, Bojórquez and Iervolino [5] and Bojórquez et al. [6] proposed an advanced scalar *IM* based in $Sa(T_1)$ and Np, called I_{Np}. In addition, Buratti [7] carried out an exhaustive comparison of the most important *IMs* available in the literature in terms of efficiency and sufficiency, concluding that the most effective intensity measure is I_{Np}. Additionally, it has been demonstrated that *IMs* resulting from the combination of $Sa(T_1)$ and Np predicts efficiently the maximum interstory drift, which is one of the most used parameters by seismic design codes to provide a suitable structural performance of earthquake-resisting structures.

Furthermore, the applicability of currently available ground motion attenuation models (*GMMs*) can be extended if the correlation between spectral acceleration values at multiple periods or orientations is known. The knowledge of these correlation coefficients is essential to perform more accurate and sophisticated *PSHA*, such as analysis with vector-valued *IMs* [8] or advanced scalar *IMs*. Recently, the advantages of the so-called *conditional mean spectrum* have been investigated, it is argued as a useful tool for ground-motion selection as input to dynamic analysis [9], and the correlation coefficients have a fundamental role in the determination of that spectrum. Inoue and Cornell [10] developed an equation to predict the correlation between spectral velocity values at different periods, with the objective of quantifying the damage in systems of multiple degree of freedom systems through an equivalent single degree of freedom system. On the other hand, Cordova *et al*. [1] presented a methodology to evaluate the seismic collapse performance of frame structures. The procedure included an *IM* that combines the $Sa(T_1)$ and a parameter which try to account for structural “softening”; for this, it was necessary correlate spectral acceleration values at two periods, and the equation proposed by Inoue and Cornell was applied. In addition, Baker and Cornell [11] using ground motions recorded in California, developed approximate analytical equations to predict the correlation between spectral acceleration values for a ground motion component at two different periods. Baker and Jayaram [12] using the *GMMs* derived from the *NGA* project, presented new refined and complex equations to predict the correlation between spectral acceleration values at two vibration periods. Jayaram and Baker [13] investigated whether the model proposed by themselves in 2008 was appropriate to predict the correlations obtained using Japanese ground motions records. They observed differences in the expected correlation values, attributing it to the dependence of the characteristics of the faulting mechanisms and source-to-site distance; therefore, selecting expressions from seismic source areas with a particular style of faulting may not apply to the interest area. For example, Cimellaro [14] adapted two models proposed by other researchers with a European ground-motion database; however, these models did not adequately predict the correlation values for that particular area.

Motivated by the need to carry out seismic hazard analysis with advanced *IMs*, a *PSHA* is determined at some sites of firm soil and soft soil of the valley of Mexico using I_{Np}. For this purpose, *GMMs* derived with available data recorded at accelerometer stations installed in CU (Ciudad Universitaria) were used; those stations are located within the hill zone area (firm soil) of the valley of México. Additionally, given the necessity to account for the correlation between spectral acceleration values at different periods, interplate earthquakes recorded in station CU were compiled; subsequently, the correlation coefficients were obtained from the residuals of the spectral acceleration between a real response spectrum and a calculated one, using its corresponding attenuation function. Afterwards, based on the nonlinear least squares method, approximate analytical equations were proposed to predict the correlation between the logarithms of spectral accelerations at two vibration periods, for interplate events. Finally, with the attenuation models, correlation coefficients and a methodology presented in the present paper, the complete distribution of the logarithm of I_{Np} can be described; with this, *PSHA* is carried out (as is done for a scalar value of $Sa(T_1)$). The results are presented through hazard curves and uniform hazard spectra (*UHS*) of I_{Np}, and these are compared with their respective $Sa(T_1)$ *UHS*.

Methodology to perform a PSHA using I_{Np}

GMMs are overriding in the *PSHA*; therefore, it is crucial to have attenuation models that predict the ground motion parameter intended to define the characteristics of future earthquakes. Unfortunately, an attenuation model has not yet devised to provide I_{Np} as a function of the vibration period, as it is done with existing attenuation models; however, with tools currently available for other ground motion *IMs*, it is possible to perform a *PSHA* with the ground motion intensity measure I_{Np}, which is defined as follows:

$$I_{Np} = Sa(T_1) \cdot N_P{}^{\alpha} \tag{1}$$

$$N_P = \frac{Sa_{avg}(T_1 \ldots T_N)}{Sa(T_1)} \tag{2}$$

where I_{Np} is the scalar intensity measure, α is a parameter that should be calibrated according to the structure and the earthquake demand parameter selected (in this study a value α=0.5 is adopted, as suggested in reference [7]), and Sa_{avg} is the geometric mean of the spectral acceleration in a given range of periods; it is expressed as:

$$Sa_{avg}(T_1 \ldots T_N) = \left(\prod_{i=1}^{N} Sa(T_i) \right)^{1/N} \tag{3}$$

Substituting Eq. (2) and Eq. (3) in Eq. (1) and applying the natural logarithm, it results:

$$\ln\left(I_{Np}\right) = (1-\alpha)\ln\left[Sa\left(T_1\right)\right] + \frac{\alpha}{N}\sum_{i=1}^{N}\ln\left[Sa\left(T_i\right)\right] \tag{4}$$

Then, the expected value and the variance of the $\ln(I_{Np})$ in Eq. (4) can be expressed as in Eq. (5) and Eq. (6), respectively.

$$E\left[\ln\left(I_{Np}\right)\right] = (1-\alpha)E\left\{\ln\left[Sa\left(T_1\right)\right]\right\} + \frac{\alpha}{N}\sum_{i=1}^{N}E\left\{\ln\left[Sa\left(T_i\right)\right]\right\} \tag{5}$$

$$Var\left[\ln\left(I_{Np}\right)\right] = \alpha^2 Var\left\{\ln\left[Sa_{avg}\left(T_1..T_N\right)\right]\right\} + \left(1-\alpha\right)^2 Var\left\{\ln\left[Sa\left(T_1\right)\right]\right\}$$
$$+ 2\alpha(1-\alpha)\rho_{\ln\left[Sa_{avg}\left(T_1...T_N\right)\right],\ln\left[Sa\left(T_1\right)\right]}\sigma_{\ln\left[Sa_{avg}\left(T_1...T_N\right)\right]}\sigma_{\ln\left[Sa\left(T_1\right)\right]} \tag{6}$$

The $\ln[Sa(T_i)]$ terms in the above equations are obtained from existing attenuation models, and because the $\ln[Sa(T_i)]$ terms are commonly assumed with a joint normal distribution, consequently, the summation has also a normal distribution. Therefore, the variance $Var\{\ln[Sa_{avg}(T_1\ldots T_N)]\}$ and the correlation coefficient $\rho_{\ln[Sa_{avg}(T_1\ldots T_N),\ln[Sa(t1)]}$ can be obtained by Eqs. (7) and (8), respectively:

$$Var\left\{\ln\left[Sa_{avg}\left(T_1..T_N\right)\right]\right\} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\rho_{\ln\left[Sa\left(T_i\right)\right],\ln\left[Sa\left(T_j\right)\right]}\sigma_{\ln\left[Sa\left(T_i\right)\right]}\sigma_{\ln\left[Sa\left(T_j\right)\right]}\right] \tag{7}$$

$$\rho_{\ln\left[Sa_{avg}\left(T_1...T_N\right)\right],\ln\left[Sa\left(T_1\right)\right]} = \frac{\sum_{i=1}^{N}\rho_{\ln\left[Sa\left(T_i\right)\right],\ln\left[Sa\left(T_1\right)\right]}\sigma_{\ln\left[Sa\left(T_i\right)\right]}}{\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\rho_{\ln\left[Sa\left(T_i\right)\right],\ln\left[Sa\left(T_j\right)\right]}\sigma_{\ln\left[Sa\left(T_i\right)\right]}\sigma_{\ln\left[Sa\left(T_j\right)\right]}\right]}} \tag{8}$$

where $\rho_{\ln[Sa(Ti)],\ \ln[Sa(Tj)]}$ is the correlation between spectral acceleration values at periods T_i and T_j. The correlations have a key role to perform probabilistic seismic hazard analyses with vector-valued and advanced scalar intensity measures, among other applications. Thus, it has been proposed an attenuation model for I_{Np}, and all the equations above are enough to describe the complete distribution of I_{Np}. The only issue that concerns is to have the correlation coefficients between spectral acceleration values. In the following sections, it is explained how those correlations are obtained, in addition, predictive mathematical expressions are presented, corresponding to interplate seismic events.

Determination of correlation coefficients

First, to obtain $\rho_{\ln[Sa(Ti)],\ \ln[Sa(Tj)]}$ it is required to have a reliable ground motion database of the area of interest. Here, we use exclusively ground motions from interplate events recorded at the accelerometer stations in CU, which are located within the hill zone area (firm ground). To adequately describe the determination of the correlation functions, it is useful to note that an attenuation function has the following form:

$$\ln Sa(T) = \mu_{\ln Sa}(M,R,\theta,T) + \sigma_{\ln Sa}(T)\varepsilon(T) \tag{9}$$

where $\mu_{\ln Sa}(M,R,\theta,T)$ and $\sigma_{\ln Sa}(T)$ are the predicted mean and the standard deviation of the natural logarithm of spectral acceleration at a specified period (T) given by the attenuation model, as a function of earthquake magnitude (M), source-to-site distance (R) and other parameters, (θ). Rearranging Ec. (9) for $\varepsilon(T)$, it results:

$$\varepsilon(T) = \frac{\ln Sa(T) - \mu_{\ln Sa}(M,R,\theta,T)}{\sigma_{\ln Sa}(T)} \tag{10}$$

where $\varepsilon(T)$ represents the number of standard deviations by which the actual logarithmic spectral acceleration differs from the predicted mean value $\mu_{\ln Sa}(M,R,\theta,T)$. For a given ground motion with known values of $Sa(T)$, M, R, etc., $\varepsilon(T)$ is also a known value. The values of $\varepsilon(T)$ at different periods are probabilistically correlated. For instance, if a recorded spectral acceleration is greater than the expected value (i.e., $\varepsilon(T)$ larger than 0) at a specified vibration period, then it is likely to be also greater than that expected at adjacent periods [14]. This relation can be characterized through correlation coefficients between ε's, as a function of two periods of interest.

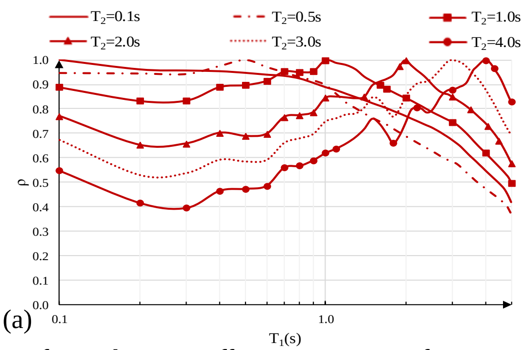
The correlation coefficient between two sets of observed ε values can be estimated using the Pearson correlation coefficient, which estimates the correlation coefficient between $\varepsilon(T_1)$ and $\varepsilon(T_2)$ as follows:

$$\rho_{\varepsilon(T1),\varepsilon(T2)} = \frac{\sum_{i=1}^{n}\left(\varepsilon_i(T_1)-\overline{\varepsilon(T_1)}\right)\left(\varepsilon_i(T_2)-\overline{\varepsilon(T_2)}\right)}{\sqrt{\sum_{i=1}^{n}\left(\varepsilon_i(T_1)-\overline{\varepsilon(T_1)}\right)^2\sum_{i=1}^{n}\left(\varepsilon_i(T_2)-\overline{\varepsilon(T_2)}\right)^2}} \tag{11}$$

where $\varepsilon_i(T_1)$ and $\varepsilon_i(T_2)$ are the *i*th observations of $\varepsilon(T_1)$ and $\varepsilon(T_2)$; $\overline{\varepsilon(T_1)}$ and $\overline{\varepsilon(T_2)}$ are the average value of the set, and *n* is the number of records. This calculation is repeated for each pair of periods of interest. The resulting correlations can be tabulated, and use them when are needed, however, it would be complicated due to the number of values that the possible table would have, for this reason, here analytical predictive mathematical expressions are fitted to the correlation coefficients.

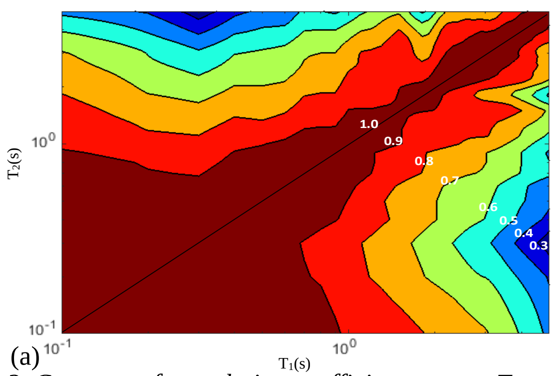
Observed correlations and predictive equations

The correlation coefficients obtained from the Reyes *et al*. [15] model, for a selection of a pair of periods, are shown in Fig. 1a. Meanwhile, Fig. 2a shows the same results; these are plotted using the correlation coefficients contours as a function of both T_1 and T_2.



(a)

Figure 1. Plots of correlation coefficients versus T_1, for several T_2 values.



(a)

Figure 2. Contours of correlation coefficients versus T_1 and T_2.

Finally, the predictive equation is the following:

$$\rho\ \ln[Sa(T_i)]\,,\ln[Sa(T_j)] = \frac{a + bTmin + cTmax}{1 + dTmin + eTmax} - fln\left(\frac{Tmax}{Tmin}\right) \tag{12}$$

where T_{min}=min(T_1,T_2) and T_{max}=max(T_1,T_2); the numerical coefficients *a, b, c, d, e* and *f* are in Table 1. These equations are valid when T_1 and T_2 are between 0.1s and 5.0s. The form of the equations has no physical meaning; it is just a fit to the observed data; therefore, they should not be extrapolated to other conditions. Fig. 3a and Fig. 3b show the correlation coefficients achieved with the predictive equation.

Table 1. Predictive equation to determine correlations.

Restriction	*a*	*b*	*c*	*d*	*e*	*f*
$T_{min} < 0.2$s	0.9881	0.3988	0.0126	0.0921	0.3126	-0.0275
$T_{min} \leq 1.5$s and $T_{min} > 0.3$s and $T_{max} \geq 1.6$s	1.0494	0.8771	-0.1968	0.9700	-0.1667	0.1694
$T_{min}=0.1$ or $T_{min} \leq 0.3$s and $T_{max} > 1.1$s	1.2438	-0.7038	-0.1415	0.5173	-0.0920	0.0813

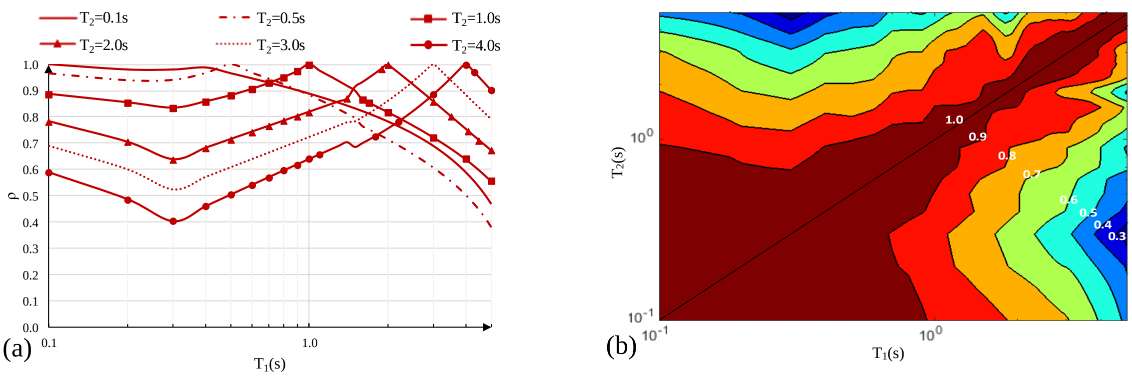


Figure 3. (a) Correlation coefficients versus T_1, for several T_2 values and (b) Contours of correlation coefficients versus T_1 and T_2, using equation 1.

Application of the predictive equations using I_{Np}

Currently, the design spectra available in earthquake-resistant design codes around the world are established, among other things, using uniform hazard spectra (*UHSs*). However, they do not take into account the cumulative plastic demands or the particularities of the hysteretic cycles when a structure undergoes to non-linear behavior. Therefore, in this study, *UHS*s were computed regarding I_{Np} and $Sa(T_1)$, for two zones located on the firm ground and soft soil of Mexico City, named as zone A and zone B, respectively. The *PSHA* was carried out employing a specific seismic regionalization for small, moderate and characteristics seismic events ($M_w > 7$).

Uniform hazard spectrum for hard soil

In first place, the uniform hazard spectrum for the CU station is obtained, which will serve as a reference to proceed with a technique based on response spectral ratios to estimate the *UHS* in a particular soft soil zone of Mexico City. The technique is described in the next section. Fig. 4a and Fig. 4b show the *UHSs* in terms of $Sa(T_1)$ and I_{Np} for CU station; which has a dominant soil period between 0.2 and 0.3s. In Fig. 4a, the *UHS*s are presented in such a way that only interplate or, alternatively, intraslab earthquakes could occur. On the other hand, Fig. 4b shows the total $Sa(T_1)$ and I_{Np} *UHS*s (considering both types of events). It is observed in Fig. 4b that both spectra $Sa(T_1)$ and I_{Np} are quite similar, practically, they reach the same acceleration levels, and visible differences occur for long periods.

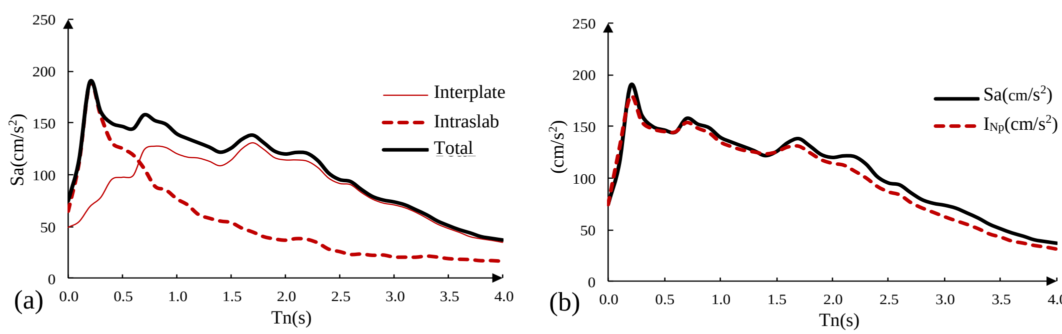


Figure 4. (a) $Sa(T_1)$ uniform hazard spectra for CU station (interplate and intraslab events) and (b) $Sa(T_1)$ and I_{Np} uniform hazard spectra for CU station.

Uniform hazard spectrum for soft soil

In this section, the procedure used to estimate the *UHS*s for soft soil is briefly described. Through a probabilistic hazard analysis is possible to evaluate ground motion hazard curves associated with different periods, and then to generate an *UHS*. This is the way the *UHS*s above were computed, and it was possible because *GMM*s for CU station were available. Nevertheless, when there is not *GMM*s available for the site of interest, a conventional *PSHA* cannot be performed. Hence, Esteva [16] (Eq. (13)) presented a formulation in which through a known hazard curve at a given site it is possible to estimate a hazard curve in another, as long as there are enough seismic events recorded simultaneously at both the reference site and the recipient site. The above is achievable by coupling this formulation with the *response spectral ratios* (*RSR*), which are the ratios between acceleration response spectra corresponding to soft soil and firm ground the ratios represent approximately the spectral amplification in soft soil with respect to firm ground. Here, the CU station is considered as the reference site. Finally, through the hazard curves from CU and the response spectral ratios in terms of $Sa(T_1)$ and I_{Np}, the *UHS*s from different accelerometer stations are estimated as follows [16]:

$$\nu_Y(y) = \int_0^{\infty} \nu_X\left(\frac{y}{z}\right) f_z(z)dz = E_z\left(\nu_x\left(\frac{y}{z}\right)\right) \tag{13}$$

where:

$\nu_Y(y)$ is the mean annual rate of exceedance of a seismic *IM* from the recipient site.

$\nu_X(y/z)$ is the mean annual rate of exceedance of a seismic *IM* from the reference site divided by the variable z.

z is the acceleration response spectral ratio (Y/X).

$f_Z(z)$ is the probability density function of the aleatory variable z.

Fig. 5a and Fig. 5b show the uniform hazard spectra of $Sa(T_1)$ and I_{Np}, for firm ground and soft soil, respectively. It is observed that the spectral ordinates of the *UHS* for hard soil have comparable acceleration values for both seismic *IM*: $Sa(T_1)$ and I_{Np}. However, it is noticed that for soft soil, the spectral ordinates corresponding to the I_{Np} uniform hazard spectrum are higher than the $Sa(T_1)$ UHS for vibration structural periods smaller than the dominant period at the site. On the contrary, lower acceleration values are reached for structural periods higher than the dominant soil period.

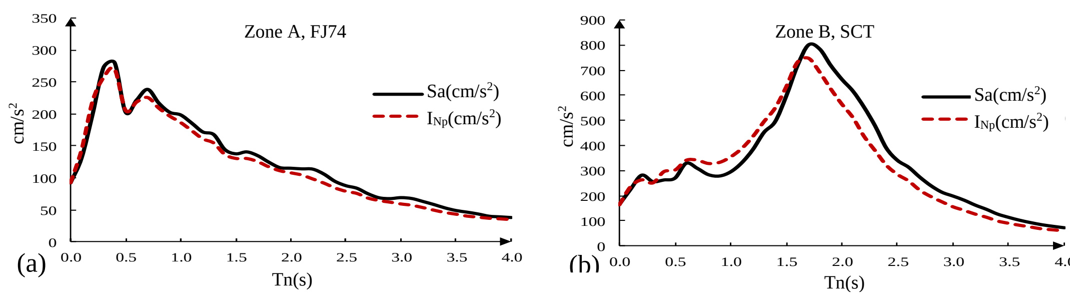


Figure 5. Uniform hazard spectra (250-year return period) for: (a) FJ74 (firm ground) and (b) for SCT (soft soil).

CONCLUSIONS

A mathematical expression to predict the correlation coefficients between spectral acceleration values at multiple periods corresponding to interplate earthquakes occurring in Mexico was proposed. The equation may have applications related to hazard analysis. Here, a *PSHA* was performed for two sites of Mexico City, using a new ground motion intensity measure called I_{Np}. Additionally, the *UHS*s corresponding to the sites located on firm ground and soft soil were computed in terms of I_{Np} and $Sa(T_1)$. It was observed that the maximum values of spectral amplification of the of I_{Np} UHS with respect to the $Sa(T_1)$ UHS occur for structures with vibration periods shorter than the dominant soil period. However, for structural periods larger than the dominant period, the structural softening produces a beneficial effect (because the lateral strength requirements are lower than those for a structure that does not take into account the period elongation). This benefit is not the same for all type of soils; it depends on the bandwidth of the ground motions; for this reason, new mathematical expressions associated with different kinds of soil are being obtained.

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REFERENCES

[1] Cordova, P. P., Dierlein, G. G., Mehanny, S. S. and Cornell, C. A. (2001). “Development of a two-parameter seismic intensity measure and probabilistic assessment procedure”, *The second U.S.-Japan Workshop on Performance-Based Earthquake Engineering Methodology for Reinforce Concrete Building Structures*, Sapporo, Hokkaido 2001, 187–206.

[2] Baker, J. W. and Cornell, C. A. (2005) “A Vector-Valued Ground Motion Intensity Measure Consisting of Spectral Acceleration and Epsilon”. *Earthquake Engineering and Structural Dynamics*, 34(1), 1193–1217.

[3] Tothong, P. and Luco, N. (2007) “Probabilistic Seismic Demand Analysis Using Advanced Ground Motion Intensity Measures”. *Earthquake Engineering and Structural Dynamics*, 36(1), 1837–1860.

[4] Bojórquez, E., Iervolino, I. and Manfredi, G. (2008). “Evaluating a new proxy for spectral shape to be used as an intensity measure”. *In: Proceedings of the 2008 seismic engineering conference commemorating the 1908 Messina and Reggio Calabria earthquake* (MERCEA ’08).

[5] Bojórquez, E., and Iervolino, I. (2011). “Spectral shape proxies and nonlinear structural response”. *Soil Dynamics and Earthquake Engineering*, 31, 996–1008.

[6] Bojórquez, E., Iervolino, I., Reyes-Salazar, A. and Ruiz, S. E. (2008). “Comparing vector-valued intensity measures for fragility analysis of steel frames for the case of narrow-band ground motions”, *Engineering Structures*, 45, 472–480.

[7] Buratti, N. (2011). "Confronto tra le performance di diverse misure di intensità dello scuotimento sísmico". *Congreso Nacional de Ingeniería de Italia*, ANIDIS, Bari Italia.

[8] Bazurro, P. and Cornell, C. A. (2002). “Vector-valued probabilistic seismic hazard analysis”. *In 7th U.S. National Conference on Earthquake Engineering*, Boston, Massachusetts, 21–25 July 2002, 10 pp.

[9] Baker, J. W. (2011). “Conditional Mean Spectrum: Tool for Ground-Motion Selection”. *Journal of Structural Engineering, ASCE*, 137(3), 322–331.

[10] Inoue, T. and Cornell, C. A. (1990). “*Seismic Hazard Analysis of Multi- Degree-of-Freedom Structures”*. Reliability of Marine Structures, RMS-8, Stanford, California, 70 pp.

[11] Baker, J. W. and Cornell, C. A. (2006). “Correlation of Response Spectral Values for Multicomponent Ground Motions”. *Bulletin of the Seismological Society of America*, 96(1), 215–227.

[12] Baker, J. W. and Jayaram, N. (2008). “Correlation of Spectral Acceleration Values from NGA Ground Motion Models”. *Earthquake Spectra*, 24(1),299–317.

[13] Jayaram, N., Baker, J. W., Okano, H., Ishida, H., McCann Jr., M. W. and Mihara, Y. (2011). “Correlation of response spectral values in Japanese ground motions”. *Earthquakes and Structures*, 2(4), 357–376.

[14] Cimellaro, G. P. (2013). “Correlations in spectral accelerations for earthquakes in Europe”. *Earthquake Engineering and Structural Dynamics*, 42, 623–633.

[15] Reyes, C., Miranda, E., Ordaz, M. and Meli, R. (2002). “Estimación de espectros de aceleraciones correspondientes a diferentes periodos de retorno para las distintas zonas sísmicas de la ciudad de México”. *Revista de Ingeniería Sísmica*, 66(1), 95–121.

[16] Esteva, L. (1970). “Regionalización sísmica de México para fines de ingeniería”, Serie Azul 246, Instituto de Ingeniería, UNAM.